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Solution Method for Countercurrent Plug Flow Models of Multicomponent Gas Separation by Permeation

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Abstract: A simple solution method was developed for the countercurrent ideal plug flow model of multicomponent gas separation by permeation that is commonly applied to hollow fiber membrane modules. The solution method is independent of the number of components in the feed gas. The result is an implicit function in a single variable defined as the stage cut. The function is readily solved by the method of damped successive substitution. The method was tested for three cases from the literature and found to be insensitive to the initial guess for stage cuts less than 60%.

Keywords: Countercurrent plug flow model, gas separation, permeation, hollow fiber membrane modules

INTRODUCTION

Multicomponent gas separation by permeation is finding industrial applications ranging from petrochemical processing to air pollution control (1, 2). Despite the potential for large pressure drop through the fiber bore, hollow fiber membranes are commonly used for gas permeators because they provide a large contact area with excellent membrane structural support in a compact module (3). The design of hollow fiber membrane modules is similar to that for a shell and tube heat exchanger. A hollow fiber membrane module consists of a bundle of membrane fibers

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mounted in a cylindrical shell. A high pressure feed gas may be introduced to either the shell or bore side of the hollow fibers. The components of the gas mixture permeate through the membrane to the low pressure side at different rates to affect a separation. In a hollow fiber module, the gases on the feed and permeate sides of the membrane move in parallel flow. The non permeating gas or residue stream is collected at the opposite end from the feed stream. It has been demonstrated that the countercurrent flow pattern, shown in Fig. 1, provides the most efficient separation in terms of minimizing the required membrane area (3). In countercurrent flow, the low-pressure permeate gas stream exits the module at the feed end. The permeate stream is typically closed at the residue end of the module. However, a sweep gas may be introduced to the permeate side to enhance the separation by decreasing the partial pressure of the permeating gas species (4). A permeate sweep stream was not considered further in this work.

The ratio of the permeate stream to feed stream molar flow rates is the stage cut.

$$\theta = \frac{n_P}{n_F} \tag{1}$$

The rejection is the ratio of the residue stream to feed stream molar flow rates.

$$1 - \theta = \frac{n_R}{n_F} \tag{2}$$

Mathematical models assuming ideal plug flow patterns have been shown to describe the performance of hollow fiber membrane modules in terms of the stage cut and exit stream compositions (3–5). In plug flow, the compositions and pressure of the parallel gas streams change in the axial direction only. Plug flow models consist of differential equations for the material balances that include the total and individual species molar flow rates and pressure change along the membrane.

The solution to the model equations for concurrent flow is relatively straightforward because the operating conditions for the feed and permeate stream are specified at the same end of the membrane module. However, mathematical models of countercurrent flow are complicated by split boundary conditions. Typically, the feed rate and composition is specified at the feed end of the membrane module, where the permeate stream exiting flow rate and composition are unknown. At the opposite end the permeate stream conditions are specified in terms of the unknown residue stream

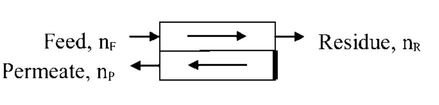

Figure 1. Countercurrent flow pattern in a membrane module with closed permeate end: n_F, n_R, and n_P are the molar feed, residue, and permeate rates, respectively.

composition. This complication has led several authors to develop a variety of solution methods to treat the split boundary value problem for the countercurrent flow pattern.

Early treatments applied variations of the iterative shooting method. A guess for the unknown boundary conditions at one end of the module is upgraded to match the solution to the differential material balances with the specified conditions at the opposite end of the module. Wegstein's (6) and Powell's (3, 6, 7) methods have been used to upgrade the guesses. This approach is highly sensitive to the selection of the guessed conditions required to initiate the solution procedure (6, 7). The solution method becomes increasingly sensitive as the number of components in the feed mixture is increased as does the computational requirement.

Coker, et al. (4) applied the tanks-in-series model to solve for the pressure and composition profiles simultaneously. They treated the residue and permeate streams as a series of well-mixed stages. The model consists of a set of material and energy balances around each well mixed stage. They solved the resulting system of nonlinear algebraic equations simultaneously for the species molar flow rates throughout the membrane module. Their solution method requires a large set of initial guesses for the compositions and flow rates from each stage throughout the permeator. This approach is essentially a first order implicit Euler integration scheme. The tanks-in-series solution method is less sensitive to initial guesses for the compositions and flow rates, but is computationally less efficient, requiring a large number of stages to give the same accuracy of higher order integration schemes that use fewer stages.

Other solution methods make restrictions on the specifications or other simplifying assumptions. Pan (5) developed an analytical solution for the special case where the residue composition of one of the species in the mixture is specified. The resulting algebraic expressions require an iterative solution method. Kovvali, et al. (8) developed analytical expressions for the flow rates and composition, and pressure profiles in countercurrent flow modules by assuming a linear relationship between the high and low pressure stream compositions. Their results assume cross flow behavior and are independent of the flow directions. More recently, Kaldis, et al. (9) applied the method of orthogonal collocation to solve the boundary value problem. The collocation method requires an iterative solution, but was shown to be less sensitive to initial guesses required to initiate the solution method. A disadvantage of orthogonal collocation is the trade-off between the number of collocation points, solution accuracy, and computational requirements. Nevertheless, orthogonal collocation provides a powerful technique for problems with a large stage cut.

The aim of this work was to develop a simpler approach to solving the countercurrent flow model equations that avoids complex algorithms or simplifications to the model equations. The approach transforms the model equations into an implicit function for the stage cut:

$$\theta = f(\theta) \tag{3}$$

Implicit functions in the form of Equation (3) are readily solved by the relatively straightforward method of successive substitution. Although successive substitution is linearly convergent, the method is easy to implement and independent of the number of components in the feed gas mixture. The method was also found to be relatively insensitive to the initial guesses for the unknown boundary conditions of stage cut, pressure, and permeate composition.

COUNTERCURRENT PLUG FLOW MODEL

The solution-diffusion model (2) of permeation through a semi-permeable membrane describes the local flux of gas species i across the permeator:

$$J_i = Q_i(P_H x_i - P_L y_i) \tag{4}$$

where J_i is the species flux, P_H and P_L and x_i and y_i are the pressures and mole fractions of species i in the high pressure stream on the feed side and low pressure stream on the permeate side of the membrane, respectively. Q_i is the permeance of species i for the particular membrane material, which is defined as the ratio of the permeability coefficient, q_i, to membrane thickness, δ:

$$Q_i = \frac{q_i}{\delta} \tag{5}$$

As shown in Fig. 2, overall and species mole balances around the residue end of the membrane module and an arbitrary position along the membrane give

$$n_H = n_R + n_L \tag{6}$$

$$x_i n_H = x_R n_R + y_i n_L \tag{7}$$

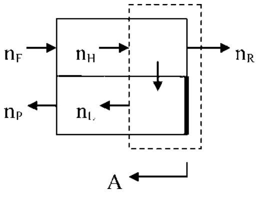

Figure 2. Mass balance around the residue end of a countercurrent flowing membrane module.

where x_i and y_i and n_H and n_L are the species mole fractions and total molar flow rates in the high and low pressure streams, respectively.

The derivation of the plug flow model equations used here follows the method of Shindo, et al. (3). However, the species mole balances are defined in terms of the permeate stream composition instead of the feed side stream composition. This model assumes an ideal plug flow pattern with constant temperature and species permeances. Resistance to mass transfer is assumed to be limited to solution and diffusion in the active layer of the membrane. The local change in the total molar flow rate of the permeate stream and molar flow rate of species i with respect to permeation area are:

$$\frac{dn_L}{dA} = \sum_{l=1}^{N_c} Q_i(P_H x_l - P_L y_l) \tag{8}$$

$$\frac{d(y_i n_L)}{dA} = Q_i(P_H x_i - P_L y_i) \quad \text{for all } i \neq j \tag{9}$$

where N_c is the number of components in the gas mixture, A is the membrane area based on the outside diameter of the fiber, and j is the reference species defined in terms of the component with the smallest permeance. The material balances are integrated from the closed end of the module to the feed end. The mole fractions in the permeate stream are subject to the constraint:

$$\sum y = 1 \tag{10}$$

The plug flow model equations may be recast in dimensionless form using the following dimensionless parameters.

$$S = \frac{AQ_j P_F}{n_F} \tag{11}$$

$$\alpha_i = \frac{Q_i}{Q_j} \tag{12}$$

$$\gamma = \frac{P_L}{P_H} \tag{13}$$

$$\phi = \frac{n_L}{n_F} \tag{14}$$

where S represents the dimensionless area of permeation, α_i is the ideal separation factor for species i relative to the reference species j, γ is the transmembrane pressure ratio, and ϕ is the local stage cut. In dimensionless terms, the plug flow model equations become

$$x_i = \frac{x_{R,i}(1-\theta) + y_i\phi}{1-\theta+\phi} \tag{15}$$

$$\frac{d\phi}{dS} = \sum_{l=1}^{N_c} \alpha_l(x_l - \gamma y_l) \tag{16}$$

$$\frac{dy_i}{dS} = \frac{\alpha_i(x_i - \gamma y_i) - y_i \sum_{l=1}^{N_c} \alpha_l(x_l - \gamma y_l)}{\phi} \quad \text{for all } i \neq j \tag{17}$$

$$y_j = 1 - \sum_{i=1(\neq j)}^{N_c} y_i \tag{18}$$

At the closed end of the permeate stream where the flow is zero, Equation (17) reduces to

$$\frac{dy_i}{dS} = 0 \quad \text{for all } i \text{ at } S = 0 \tag{19}$$

The change in pressure is calculated from the Hagen-Poiseuille equation for laminar flow in hollow fibers.(4, 5, 9)

$$\frac{d\gamma}{dS} = -\frac{\phi\beta}{\gamma} \tag{20}$$

where

$$\beta = \frac{128\mu n_F^2 R_g T}{N_f^2 \pi^2 D_i^4 D_o P_F^3 Q_j} \tag{21}$$

and where R_g is the ideal gas constant, T is the temperature, P_F is the pressure of the feed stream, N_f, is the number of hollow fibers in the module and D_i, and D_o are the inside and outside hollow fiber diameters, respectively. The dimensionless pressure is subject to the following boundary conditions.

$$\frac{d\gamma}{dS} = 0 \quad \text{at } S = 0 \tag{22}$$

$$\gamma = \gamma_F \quad \text{at } S = S_F \tag{23}$$

The composition of the permeate stream at the closed end of the fibers is determined by combining Equations (17) and (19), then solving for $y_{0,i}$:(3, 10)

$$y_{0,i} = \frac{\alpha_i(x_{R,i} - \gamma_0 y_{0,i})}{\sum_{l=1}^{N_c} \alpha_l(x_{R,l} - \gamma_0 y_{0,l})} \tag{24}$$

where the subscript 0 indicates the permeate stream conditions at the closed end. The ratio of $y_{0,i}$ to $y_{0,j}$ for the reference species ($\alpha_j = 1$) gives

$$\frac{y_{0,i}}{y_{0,j}} = \frac{\alpha_i(x_{R,i} - \gamma_0 y_{0,i})}{x_{R,j} - \gamma_0 y_{0,j}} \tag{25}$$

Solve Equation (25) for $y_{0,i}$:

$$y_{0,i} = \frac{\alpha_i x_{R,i}}{\gamma_0(\alpha_i - 1) + \psi} \tag{26}$$

where ψ is the ratio of reference species (j) residue to permeate mole fractions at the closed end:

$$\psi = \frac{x_{R,j}}{y_{0,j}} \tag{27}$$

Equation (26) is substituted for y in Equation (10):

$$\sum_{l=1}^{N_c} \frac{x_{R,l}\alpha_l}{\gamma_0(\alpha_l - 1) + \psi} = 1 \tag{28}$$

Equation (28) is nonlinear in ψ and requires an iterative solution technique, such as Newton's method, to find ψ. The permeate stream composition at the closed end is then calculated from Equation (26).

CALCULATION METHOD

The solution procedure requires an initial guess for the stage cut and permeate stream composition. Chen (7) showed that the following algebraic model assuming an average driving force gives a reasonable approximation to parallel flow behavior under limiting conditions of low stage cut. The species mole balances take the following form.

$$x_{F,i} = (1 - \theta)x_{R,i} + \theta y_{P,i} \tag{29}$$

The rate of permeation is calculated using an average species composition in terms of the mole fraction in the feed and residue streams. The permeate side is assumed to be well-mixed. This simplification avoids the requirement of solving for the permeate and residue stream compositions simultaneously.

$$y_{P,i}\theta = S\alpha_i\left(\frac{x_{F,i} + x_{R,i}}{2} - \gamma_F y_{P,i}\right) \tag{30}$$

Equations (29) and (30) are solved simultaneously for the residue and permeate mole fractions.

$$x_{R,i} = \frac{x_{F,i}[2\theta - S\alpha_i(\theta - 2\gamma_F)]}{2\theta(1-\theta) + S\alpha_i[\theta + 2\gamma_F(1-\theta)]} \tag{31}$$

$$y_{P,i} = \frac{x_{F,i}S\alpha_i(2-\theta)}{2\theta(1-\theta) + S\alpha_i[\theta + 2\gamma_F(1-\theta)]} \tag{32}$$

The stage cut in Equations (31) and (32) is found by applying the following constraint, similar to the Rachford-Rice method applied to the vapor-liquid equilibrium in an ideal stage:

$$\sum x - \sum y = 0 \tag{33}$$

This results in a single nonlinear equation for the stage cut.

$$\sum_{i=1}^{N_c} \frac{x_{F,i}[2\theta - 2S\alpha_i(1-\gamma_F)]}{2\theta(1-\theta) + S\alpha_i[\theta + 2\gamma_F(1-\theta)]} = 0 \tag{34}$$

Equation (34) is readily solved by iterative techniques, such as Newton's method. The permeate and residue stream compositions are then calculated from Equations (31) and (32).

The pressure at the closed end of the permeate stream is initially approximated by integrating Equation (20) assuming a linear average of the stage cut between the permeate closed end where $\phi = 0$ and the exit end where $\phi = \theta$:

$$\gamma_0 \cong \sqrt{\gamma_F^2 + S\theta\beta} \tag{35}$$

The following steps are taken to solve the implicit countercurrent plug flow model equations for the stage cut and exit stream compositions.

1. Define the reference species, j, as the component in the feed mixture with the smallest permeance.
2. Define the dimensionless parameters S, α_i, and γ_F at the feed end of the module. Set the iteration index for successive substitution to $k = 1$.
3. Use the average driving force approximation model of Chen (7) to provide initial guesses for the overall stage cut fraction, θ_k, and permeate stream composition.
4. Calculate an approximation for the pressure drop for flow inside the fibers using the Hagen-Poiseuille equation assuming a constant stage cut taken from Step 3. The result provides the initial condition for the pressure in the fibers at the permeate closed end of the module.
5. Calculate the residue stream composition in terms of the feed composition and latest approximations for the permeate stream composition

and stage cut from mole balances around the module.

$$x_{R,i} = \frac{x_{F,i} - \theta_k y_{P,i}}{1 - \theta_k} \quad \text{for all species } i. \tag{36}$$

6. Calculate the permeate stream composition at the closed end in terms of the residue mole fractions from step 5 using Equation (26)
7. Using the latest values for the unknown boundary conditions, integrate the model equations from the closed permeate end to the feed end of the membrane module for the final stage cut at the feed end, $\theta' = \phi$ feed end pressure ratio, γ'_F, and permeate stream exit composition, y'_P
8. Check for convergence by comparing the stage cut from the previous iteration with the value calculated in Step 7. The iterations of successive substitutions are terminated when the following criterion is satisfied.

$$100\left|\frac{\theta' - \theta_k}{\theta'}\right| < \varepsilon \tag{37}$$

where ε represents a prescribed percent tolerance for convergence.

9. Upgrade the stage cut fraction, permeate stream composition, and pressure at the closed end using a weighted average of the results from step 7 and the previous iteration.

$$\theta_{k+1} = w\theta' + (1 - w)\theta_k \tag{38}$$

$$(y_{P,i})_{k+1} = wy'_{P,i} + (1 - w)(y_{p,i})_k \tag{39}$$

$$\gamma_{0,k+1} = w[\gamma_F + (\gamma_{0,k} - \gamma'_F)] + (1 - w)\gamma_{0,k} \tag{40}$$

where $w < 1$ is the weighting, or damping factor when $w < 1$.

10. Increment the iteration index to $k = k + 1$ and repeat Steps 5 through 10 successively substituting the current values for θ, γ_0 and y_P until the convergence requirement for the stage cut θ in Equation (37) is satisfied.

A similar algorithm may be used to determine the dimensionless membrane area, S, given the stage cut, θ. The plug flow model equations are rearranged for ϕ as the independent variable as follows.

$$\frac{dS}{d\phi} = \left[\sum_{l=1}^{N_c} \alpha_l(x_l - \gamma y_l)\right]^{-1} \tag{41}$$

$$\frac{dy_i}{d\phi} = \frac{\alpha_i(x_i - \gamma y_i) - y_i \sum_{l=1}^{N_c} \alpha_l(x_l - \gamma y_l)}{\phi \sum_{l=1}^{N_c} \alpha_l(x_l - \gamma y_l)} \quad \text{for all } i \neq j \tag{42}$$

$$\frac{d\gamma}{d\phi} = -\frac{\phi\beta}{\gamma\phi \sum_{l=1}^{N_c} \alpha_l(x_l - \gamma y_l)} \tag{43}$$

Equations (41), (42) and (43) are integrated over the range $0 \leq \phi \leq \theta$. The result for S, γ_0 and y_P are successively substituted for the initial guesses until convergence in S is reached.

RESULTS AND DISCUSSION

The damped successive substitution solution method for the countercurrent plug flow model was tested with three different cases of multicomponent gas mixtures in semipermeable membranes reported in the literature (3, 7, 8). In each case, the dimensionless area was specified and the implicit equations were solved for the stage cut and exit stream compositions. The solution method was implemented with the computational software Mathcad, using double precision calculations. The iterative Levenberg-Marquardt method was used to solve the nonlinear equations to estimate the stage cut and permeate stream composition required to initiate the method as well as the closed-end permeate stream compositions after each iteration of successive substitution. The differential equations were solved by a variable step fourth-order Runge-Kutta integration scheme. A damping factor of $w = 0.6$ was found to give satisfactory results for all of the calculations presented here. A percent tolerance of $\varepsilon = 10^{-4}$ was used to determine convergence with stage cut and exit stream compositions precise to the fourth decimal place. This precision was necessary to compare these results with the values reported in the literature. The reference species with the smallest separation factor was positioned as component $j = 1$ in each case.

Case 1

The damped successive substation method was first tested on a three component gas mixture and membrane system described by Shindo, et al. (3). The feed stream composition and separation factors for the three components are listed in Table 1. A constant relative pressure of $\gamma = 0.13$ was assumed.

Table 1. Separation factor and stream compositions for Case 1 (3)

Gas species	α	x_F	y_P	x_R
1	15.3	0.450	0.737	0.278
2	4.86	0.250	0.201	0.279
3	1.00	0.300	0.062	0.443

The stage cut for a dimensionless area of $S = 1$ was calculated after 16 successive substitutions for $\theta = 0.375$. The solution for the permeate and residue stream compositions is also listed in Table 1. These results are in agreement with the results reported by Shindo, et al. (3) using Powell's method.

The sensitivity of the solution method to initial guesses for θ and y_P was also tested. The method converged to the same results in Table 1 with the same number of successive substitutions for all combinations of initial estimates for $0.001 < \theta < 0.6$ and y_P. The method diverged for a dimensionless area $S > 1.6$, corresponding to a relatively large stage cut, $\theta > 0.6$ at these conditions. Decreasing the damping factor did not help the convergence for $\theta > 0.7$.

Case 2

The damped successive substitution method was applied next to the nine component gas mixture-membrane system investigated by Chen, et al. (7) The solution method was tested for $S = 1$ and a constant pressure ratio, $\gamma = 0.1$. The separation factors, feed conditions, and results for the nine component system are listed in Table 2.

The stage cut was calculated as $\theta = 0.304$. These results agree with those reported by Chen (7). Although the number of components was increased three-fold to nine for this case, only 16 successive substitutions were required for convergence, the same required for a three component gas mixture in Case 1. Tests for sensitivity to the initial guesses required to initiate the successive substitution method showed no increase in the number of iterations for $0.01 < \theta < 0.6$. The method diverged for larger stage cuts at these conditions.

Table 2. Composition and separation factor for gas streams in Case 2 (7)

Gas species	α	x_F	y_P	x_R
1	20.1	0.465	0.912	0.269
2	0.612	0.056	0.007	0.078
3	0.873	0.200	0.033	0.273
4	1.02	0.061	0.012	0.083
5	0.649	0.124	0.015	0.171
6	1.46	0.014	0.004	0.019
7	1.00	0.048	0.009	0.065
8	1.46	0.023	0.006	0.030
9	1.00	0.009	0.002	0.012

Case 3

The affect of pressure drop through a hollow fiber bore was considered in this third case using the hypothetical gas mixture and conditions of Kovvali, et al. (8). The fiber inside and outside diameters were $D_i = 2 \times 10^{-4}$ m and $D_o = 3 \times 10^{-4}$ m, respectively. The feed rate to the shell side was 1.0×10^{-4} mol/s, with a pressure of 6.08×10^6 Pa, viscosity of 1.5×10^{-5} kg/m·s, and temperature of 298 K. The permeability of the reference species was $Q_j = 3.0 \times 10^{-9}$ mol/m^2·s·Pa. The feed composition and ideal separation factors for this case are listed in Table 3.

The stage cut calculation for a pressure ratio at the feed end, $\gamma_F = 0.5$ gave $\theta = 0.238$, in agreement with those of Kovvali (8). Accounting for pressure change in the fiber increased the number of successive substitutions to 19 in this case. Otherwise, the successive substitution method was tested and found to be insensitive to the initial guesses required to initiate the solution method. However, the convergence was not achieved for stage cuts greater than 0.6 for these conditions, as with the other cases studied here.

The solution for the dimensionless pressure profile is plotted in Fig. 3, along with the mole fractions of the fast species in the residue and permeates streams. The pressure change in the permeate stream was approximately 30%. The relatively large change in the dimensionless pressure resulted in stiffer differential equations that required more integration steps. Nevertheless, converged solutions were reached in a matter of seconds on a personal computer.

The lack of sensitivity to the starting conditions makes the damped successive substitution an attractive alternative to previously reported solution techniques. The method is simple to implement using standard numerical methods for solving single nonlinear algebraic equations and systems of ordinary differential equations.

In the cases shown here, higher stage cuts would result in very small mole fractions for the fast gas species in the residue. This tended to cause the method of successive substitution to diverge. The lack of convergence for large stage cuts limits the range of usefulness of the successive substitution method presented here. However, membrane modules are scaled up by adding stages. This technique should apply to staged processes that avoid

Table 3. Composition and separation factor of feed species for Case 3 (8)

Gas species	α	x_F	y_P	x_R
1	10.0	0.100	0.251	0.053
2	5.00	0.200	0.331	0.159
3	2.50	0.200	0.197	0.201
4	1.00	0.500	0.221	0.587

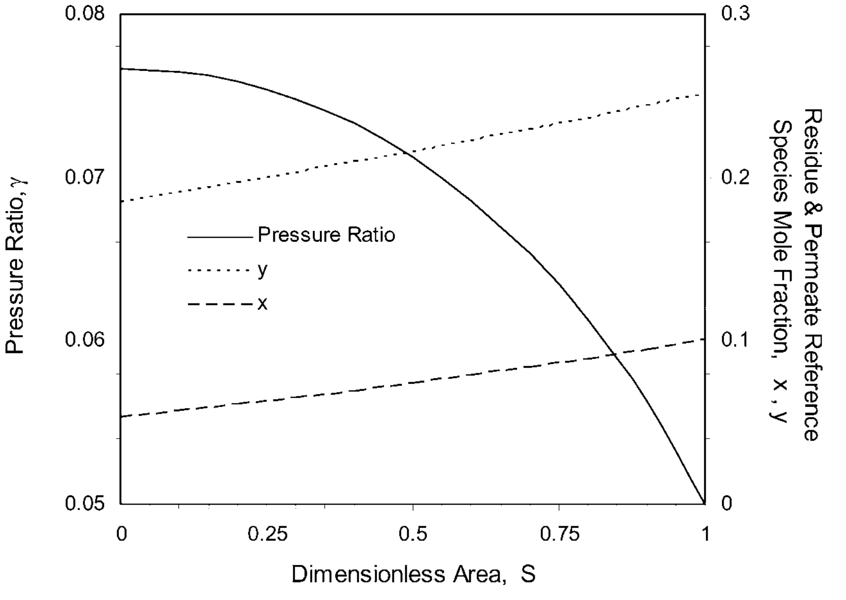

Figure 3. Profiles of pressure ratio, γ, and fast species compositions for Case 3. Closed permeate end is at $S = 0$.

large stage cuts in any one module. The method of orthogonal collocation is recommended when larger stage cuts are required.

CONCLUSIONS

A simple solution method was developed for the countercurrent ideal plug flow model of multicomponent gas separation by permeation. The model equations were arranged in the form of an implicit function for the stage cut that is readily solved by the method of damped successive substitution. The exit stream compositions are also calculated in this process. The solution method is independent from the number of components in the gas feed mixture. The damped successive substitution method was tested and found to converge for stage cuts less than 60% for the conditions of three case studies. The range of stage cut for other systems may be different. The method was found to be insensitive to the initial guesses for the unknown boundary conditions required to initiate the iterations. Pressure drop in the permeate stream was also incorporated into the model for hollow fiber membranes. The damped successive substitution method may also be used to determine the area of permeation for a given stage cut. The successive substitution is an attractive alternative because it avoids complex programming or limiting assumptions of previous solution methods and is simple to implement with computational software.

NOMENCLATURE

A	total membrane permeation area based on fiber outside diameter (m^2)
D	hollow fiber diameter (m)
J	molar flux ($mol/m^2 \cdot s$)
n	molar flow rate, (mol/s)
N_c	number of components in feed gas
N_f	number of hollow fibers
P	pressure (Pa)
q	permeability coefficient ($mol/s \cdot Pa \cdot m$)
Q	permeance ($mol/s \cdot Pa \cdot m^2$)
R_g	ideal gas constant ($8.314\ Pa \cdot m^3/mol \cdot K$)
S	dimensionless area
T	temperature, K
w	relaxation factor
x	residue or high pressure stream mole fraction
y	permeate or low pressure stream composition

Greek Symbols

α	ideal separation factor, ratio of permeability of species i to reference species j.
β	dimensionless pressure term for Hagen-Poiseiulle equation
γ	pressure ratio
δ	membrane thickness (m)
ε	percent tolerance of convergence
θ	stage cut fraction
μ	gas viscosity ($kg/m \cdot s$)
ϕ	local stage cut
ψ	ratio of residue to permeate composition of reference species at the closed end

Subscripts

0	closed end of permeate stream
F	feed gas
H	high pressure stream
i	species index
j	reference species
L	low pressure stream
P	permeate gas
R	residue gas

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